QUASI - n - BINORMAL OPERATORS

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Abstract : In this paper, we investigate some basic properties of quasi – n – binormal operators acting on a Hilbert space H. Further we study quasi – n – binormal Composite integral operators.

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1 Introduction

Let H be a Hilbertspace L(H) be the algebra of all bounded linear operators acting on H. An operator

$T \in L(H)$ is called normal if $T*T = TT^*$ n – normal if $T^*T^n = T^nT^*$ binormal if T^*T commute with

TT^*. Isometry if $T^*T = I$.

2 – isometry if $T^{*2}T^2 - 2T^*T + I = 0$

3 isometry if $T^{*3}T^3 - 3T^{*2}T^2 + 3T^*T - I = 0$

n – isometry if if $\sum_{k-0}^{n}(-1)^k \binom{n}{k} T^{*n-k}T^{n-k} = 0$

(or)

$$T^{*n}T^n - \binom{n}{1}T^{*n-1}T^{n-1} + \binom{n}{2}T^{*n-2}T^{n-2} + \ldots\ldots\ldots\ldots + (-1)^{n-1}\binom{n}{n-1}T^*T + (-1)^n I = 0$$

n – binormal if if $T^*T^nT^nT^* = T^nT^*T^*T^n$ .Quasi n – binormal if

$T(T^*T^nT^nT^*) = (T^*T^nT^nT^*)T$

Theorem 1.1 If $T \in [QnBN]$ then so are

1) KT far any real number k
2) Any $s \in L(H)$ is unitarily equivalent to T.
3) The restriction T/M of T to any closed subspace M of H that reduces T.

Proof: 1. The proof is straight forward.

2. Let $s \in L(H)$ be unitarily equivalent to T, then there is a unitary operator $u \in L(H)$

Such that $S = UTU^* \Rightarrow S^* = UT^*U^*, S^n = UT^nU^*$

If T is quasi – n – binormal then $T(T^*T^nT^nT^*) = (T^*T^nT^nT^*)T$

Consider $S^*S^nS^nS^* = UT^*U^*UT^nU^*UT^nU^*UT^*U^* = UT^*T^nT^nT^*U^*$

$S(S^*S^nS^nS^*) = UTU^*(UT^*U^*UT^nU^*UT^nU^*UT^*U^*)$ = $UTT^*T^nT^nT^*U^*$ ....(1)

$(S^*S^nS^nS^*)S = (UT^*U^*UT^nU^*UT^nU^*UT^*U^*)UTU^* = UT^*T^nT^nT^*TU^* = UT^*T^nT^nT^*TU^*$...............(2)

Hence S is unitarily equivalent to T.

4) If T is quasi n – binormal the (T/M)$\in$ (Quasi nBN)

Theorem 1.2 If $T \in L(H)$ is n – Binormal then T$\in$ (Quasi nBN)

Proof: If T is n – binormal, then $T^*T^nT^nT^* = T^nT^*T^*T^n$

Post multiply by T

$T(T^*T^nT^nT^*) = (T^nT^*T^*T^n)T$

Hence T is quasi n – binormal.

Theorem 1.3 *Let T*$\in (QnBN)$ *and S*$\in (QnBN)$**.** *If T and S are doubly commuting then TS is quasi n Binormal.*

Proof:

$(Ts)(TS)^*(TS)^n(TS)^n(TS)^* = (TS)S^*T^*S^nT^nS^nT^nS^*T^*$

$$= STT^*S^*T^nS^nT^nS^nT^*S^*$$

$$= STS^nS^*T^*T^nT^nT^*S^*S^n$$

$$= SS^*TS^nT^*T^nT^nT^*S^*S^n$$

$$= S^*S^nSTT^*T^nT^nT^*S^*S^n$$

$$= S^nS^*TST^nT^*T^*T^nS^nS^*$$

$$= S^nT^nS^*T^*S^nT^nS^*T^*ST$$

$$= (S^*T^*)(S^nT^n)(S^nT^n)(S^*T^*)(TS)$$

$$= (TS)^*(TS)^n(TS)^n(TS)^*(TS)$$

Hence TS is quasi - n – binormal.

Theorem 1.4 : *Let S andT be commuting (QnBN) operators such that* $(S+T)^n$ *commute with*

$\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k$. Then (S+T) is n – binormal operator.

Proof

$$(S+T)(S+T)^*(S+T)^n(S+T)^n(S+T)^* = (S+T)\left((S+T)^*\sum_{K=0}^{N}\binom{n}{k}S^{n-k}T^k\right)\left(\sum_{k=0}^{n}\binom{n}{k}S^{n-k}T^k(S+T)^*\right)$$

$$= (S+T)\left((S+T)^*S^n+(S+T)^*\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+(S+T)^*T^n\right)\left(S^n(S+T)^*+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k(S+T)^*+T^n(S+T)^*\right)$$

$$(S+T)\left((S^*+T^*)S^n+(S+T)^*\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^{k-}+(S^*+T^*)T^n\right)\left(S^n(S^*+T^*)+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k(S+T)^*+T^n(S^{*+}T^*)\right)$$

$$(S+T)\left(S^nS^*+T^*S^n+(S+T)^*\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+S^*T^n+T^*T^n\right)\left(S^nS^*+S^nT^*+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k(S+T)^*+T^nS^*+T^nT^*\right)$$

Since S and T are commuting Quasi n Binormal operators such that $(S+T)^*$ *commute with*

$$\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^{k}.$$

=

$$(S+T)(S^nS^*+S^nT^*+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k(S+T)^*+T^nS^*+T^nT^*)\left(S^*S^n+T^*S^n(S+T)^*\sum_{K=1}^{n-1}\binom{n}{K}S^{n-K}T^K+S^*T^n+T^*T^n\right)$$

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$$=SS^nS^*+SS^nT^*+S\sum_{K=1}^{n-1}\binom{n}{K}S^{n-K}T^K(S+T)^*+ST^nS^*+ST^nT*+TS^nS^*+TS^nT^*+T\sum_{k=1}^{n-1}\binom{n}{k}$$
$$S^{n-k}T^k(S+T)^*+TT^nS^*+TT^nT^*\left(S^*S^n+T^*S^n+(S+T)^*\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+S^*T^n+T^*T^n\right)$$

$$=S\left(S^nS^*+S^nT^*+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k(S+T)^*+T^nS^*+T^nT^*\right)+T(S^*S^n+T^*S^n+$$
$$(S+T)^*\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+S^*T^n+T^*T^n)\left(S^*S^n+T^*S^n+(S+T)^*\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+S^*T^n+T^*T^n\right)$$

$$=\left(S^nS^*+S^nT^*+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k(S+T)^*+T^nS^*+T^nT^*\right)X$$
$$(S+T)\left(S^*S^n+T^*S^n+(S+T)^*\sum_{k=1}^{n-1}S^{n-k}T^k+S^*T^n+T^*T^n\right)$$

$$=\quad(S^n+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+T^n)(S^*+T^*)(S+T)(S^*+T^*)(S^n+\sum_{k=1}^{n-1}\binom{n}{k}S^{n-k}T^k+T^n)$$

$$=\quad\sum_{k=o}^{n}\binom{n}{k}S^{n-k}T^k(S+T)^*(S+T)^*\sum_{k=0}^{n}\binom{n}{k}S^{n-k}T^k(S+T)$$

$$=\left(S+T)^n(S+T)^*(S+T)^*(S+T)^n(S+T\right)$$

Hence (S+T) is quasi n – binormal operator.

Example:1.1 Let _ $T=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}$ be an operator on R^2 which is 3QBN but either 3 binormal nor normal. Let $T=\begin{pmatrix}1 & 0\\1 & -1\end{pmatrix}$ be 3 Quasi binormal operator.Then $T\left(T^3T^*T^*T^3\right)=(T^3T^*T^*T^3)T$

$$T^2=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \qquad T^3=T^2.T=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}$$

$$T^3T^*=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\1 & -1\end{pmatrix}=\begin{pmatrix}2 & -1\\-1 & 1\end{pmatrix} \quad T^*T^3=\begin{pmatrix}1 & 0\\1 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}=\begin{pmatrix}1 & 1\\1 & 2\end{pmatrix}$$

$$T\left(T^3T^*T^*T^3\right)=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}\begin{pmatrix}2 & -1\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\1 & 2\end{pmatrix}=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}$$

$$(T^3T^*T^*T^3)T=\begin{pmatrix}2 & -1\\-1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\1 & 2\end{pmatrix}\begin{vmatrix}1 & 1\\0 & -1\end{vmatrix}=\begin{pmatrix}1 & 1\\0 & -1\end{pmatrix}$$

Example 1.2 Let $S=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}T=\begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}$ be not commuting (2QBN) operators. Then ST is 2 quasi Binormal **operators.**

Solution: Let $S=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}T=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ be two operators.

Then $ST=\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $(ST)^2=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$(ST)^*=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ $(ST)\left(ST\right)^2(ST)^*(ST)^*(ST)^2=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$

$(ST)^2(ST)^*(ST)^*(ST)^2(ST)=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$.Hence ST is quasi n binormal operators.

Example: 1.3 If T is quasi Binormal operator on R^2, then T^* is also quasi Binormal operator on R^2.

Solution Consider the operator T = $\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$ Then it can be easily verified that

$T(T^*TTT^*) = \begin{pmatrix} 9 & 18 \\ 9 & -9 \end{pmatrix} = (T^*TTT^*)T$ and $T^*(TT^*T^*T) = (TT^*T^*T)T^* = \begin{pmatrix} 9 & 18 \\ 9 & -19 \end{pmatrix}$

Hence T^* is also quasi binormal operator on R^2.

Example: 1.4 $T \in B(H)$ be an invertible operator on the real Hilbert Space H. If $T = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

Then both T and T^* are quasi normal op erators. Also T and T^* are quasi binormal operators.

Solution : $T = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ and $T^* = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$ implies that $T(T^*T) = \begin{pmatrix} 4 & 0 \\ 2 & 2 \end{pmatrix} = (T^*T)T$. Hence

T is quasi normal operator. Further $T^*(TT^*) = \begin{pmatrix} 2 & 0 \\ -2 & 4 \end{pmatrix} = (TT^*)T^*$. Hence T^* is also quasi normal operator.

$T(T^*TTT^*) = \begin{pmatrix} 8 & 0 \\ 0 & 4 \end{pmatrix} = (T^*TTT^*)T$ Implies that T is quasi binormal operator.

$T^*(TT^*T^*T) = \begin{pmatrix} 4 & 0 \\ -4 & 8 \end{pmatrix} = (TT^*T^*T)T^*$ .Hence T^* is also quasi binormal operator.

Theorem: 1.5 Let $T \in L(H)$ with the Cartesian decomposition T = A+iB .Then T is binormal if and only if

$AB^3 + B^3A = A^3B + BA^3$ and $A^2BA + ABA^2 = B^2AB + BAB^2$

Proof: Since T is quasi binormal .Then $T(T^*TTT^*) = (T^*TTT^*)T$

$$T(T^*TTT^*) = (A+iB)(A-iB)(A+iB)(A+iB)(A-iB)$$

$$= (A+iB)(A-iB)(A+iB)(A^2+B^2)$$

$$= A^5 + B^2A^3 + iA^4B + iB^3A^2 + A^3B^2 + A^3B^2 + B^4A + iA^2B^3 + iB^5$$

$$(T^*TTT^*)T = (A - iB)(A + iB)(A + iB)(A - iB)(A + iB)$$

$$= (A - iB)(A + iB)(A + iB)(A^2 + B^2)$$

$$= (A - iB)(A + iB)(A^3 + AB^2 + iA^2B + iB^3)$$

$$= (A^2 + B^2)(A^3 + AB^2 + iA^2B + iB^3)$$

$$= A^5 + A^3B^2 + iA^4B + iA^2B^3 + B^2A^3 + AB^4 + iA^2B^3 + iB^5$$

It is easy to observe that T is quasi binormal if and only if (i) and (ii) are true.

Theorem : 1.6 If C_T is quasi n binormal operator on a Hilbert space and C_s is unitarily equivalent to C_T, then C_S is quasi n binormal operator.

Proof:

$\because$ C_S is unitarily equivalent to C_T then $\exists$ an unitary operator U such that $C_S = UC_TU^*$

Consider $C_S(C_S{}^*C_S{}^nC_S{}^nC_S{}^*) = (C_S{}^*C_S{}^nC_S{}^nC_S{}^*)C_s$

$$C_S(C_S{}^*C_S{}^nC_S{}^nC_S{}^*) = (UC_TU^*)((UC_TU^*)^*(UC_TU^*)^n(UC_TU^*)^n(UC_TU^*)^*)$$

$$= UC_TU^*UC_T{}^*U^*UC_T{}^nU^*UC_T{}^nU^*UC_T{}^*U^*$$

$$= UC_TC_T{}^*C_T{}^nC_T{}^nC_T{}^*U^*$$

$$= U(C_TC_T^{*2}C_T{}^{2n})U^* \text{ ....(1)}$$

$$(C_S{}^*C_S{}^nC_S{}^nC_S{}^*)C_S = ((UC_TU^*)^*(UC_TU^*)^n(UC_TU^*)^n(UC_TU^*)^*(UC_TU^*)$$

$$= UC_T{}^*U^*UC_T{}^nU^*UC_T{}^nU^*UC_T{}^*U^*UC_TU^*$$

$$= UC_T{}^*C_T{}^nC_T{}^nC_T{}^*C_TU^*$$

$$= U(C_TC_T{}^{*2}C_T{}^{2n})U^* \text{ ....(2)}$$

From(1) and(2) C_S quasi - n – binormal composition operator.

2. Quasi –n - Binormal composite integral operators

$Let(X,S,\mu)$ be a σ finite measure space and Let $\phi: X \to X$ be a non singular measurable transformation. $\mu(E)=0 \Rightarrow \mu\phi^{-1}(E)=0$ Then a Composition transformation for $1 \le p \prec \infty$

$C_\phi : L^p(\mu) \to L^p(\mu)$ is defined by $C_\phi f = fo\phi$ for every $f \in L^p(\mu)$

In case C_ϕ is continuous .We call it a composition operator induced by ϕ . C_ϕ is bounded operatorif and only if $\frac{d\mu\phi^{-1}}{d\mu} = f_0$ .This Random Nikodym derivative of the measure $\mu\phi^{-1}$ w.r.t measure μ and it is essentially bounded.

A kernel $K \in L^p(\mu x \mu)$ always induces a bounded integral operator $T_k : L^p(\mu) \to L^p(\mu)$ defined by $(T_k f)(x) = \int K(x,y) f(y) d\mu(y)$

Given a kernel k and a nonsingular measurable function $\phi: X \to X$ the composite integral operator $T_{k\phi}$ induced by (k,ϕ) is a bounded linear operator $T_k : L^p(\mu) \to L^p(\mu)$ defined by

$$(T_{k\phi} f)(x) = \int K(x,y) f(\phi(y)) d\mu(y)$$

$$= \int k_\phi(x,y) f(y) d\mu(y)$$

$$(T_{k\phi}{}^n f)(x) = \int K^n(x,y) f(\phi(y)) d\mu(y)$$

$$= \int k\phi^n(x,y) f(y) d\mu(y)$$

$$k\phi^n(x,y) = \int\int\int ....\int k_\phi(x,z_1) k_\phi(z_1,z_2)....k_\phi(z_{n-2,}z_{n-1}) K_\phi(z_{n-1,}y) dz_1 dz_2 ...dz_{n-1}$$

Theorem 2.1 Let $k_\phi \in L^2(\mu x \mu)$. Then $T_{k\phi}$ is quasi n binormal if and only if

$$\int\int\int k_\phi(x,y)) K_\phi{}^*(y,z) k_\phi^n(z,t) k_\phi{}^*(t,p) k_\phi{}^n(p,s) d\mu(y) d\mu(z) d\mu(t) d(\mu(p) d(\mu(s) =$$
$$\int\int\int k_\phi{}^n(x,y) k_\phi{}^*(y,z) k_\phi{}^*(z,t) k_\phi{}^n(t,p) k_\phi(p,s) d(\mu(y) d(\mu(z) d\mu(t) d\mu(p) d(\mu(s)$$

Proof: Suppose the condition is true. For $f, g \in L^2(\mu)$ .We have

$$\left(T_{k\phi}T_{k\phi}^{*}T_{k\phi}{}^{n}T_{k\phi}{}^{n}T_{k\phi}{}^{*} f, g\right) = \int (T_{k\phi}T_{k\phi}{}^{*}T_{k\phi}{}^{n}T_{k\phi}{}^{n}T_{k\phi}{}^{*} f)(x)\bar{g}(x)d\mu(x)$$

$$= \int\int (k_{\phi}(xy)(T_{k\phi}{}^{*}T_{k\phi}{}^{n}T_{k\phi}{}^{n}T_{k\phi}{}^{*} f)(y)d\mu(y)\bar{g}(x)d\mu(x)$$

$$= \int\int k_{\phi}(x,y)\int (k_{\phi}{}^{*}(y,z)(T_{k\phi}{}^{n}T_{k\phi}{}^{n}T_{k\phi}{}^{*} f)(z)d\mu(z)d\mu(y)\bar{g}(x)d\mu(x)$$

$$= \int\int\int k_{\phi}(xy)k_{\phi}{}^{*}(y,z)\int k_{\phi}{}^{n}(z,t)T_{k\phi}{}^{n}T_{k\phi}^{*} f)(t)d\mu(t)d\mu(z)d\mu(y)\bar{g}(x)d\mu(x)$$

$$= \int\int\int\int k_{\phi}(x,y)k_{\phi}{}^{*}(y,z)k_{\phi}{}^{n}(z,t)(\int k_{\phi}{}^{n}(t,p) f(p)d\mu(p)d\mu(t)d\mu(z)d\mu(y)\bar{g}(x)d\mu(x)$$

$$= \int\int\int\int\int k_{\phi}(x,y)k_{\phi}{}^{*}(y,z)k_{\phi}{}^{n}(z,t)k_{\phi}{}^{n}(t,p)\int k_{\phi}{}^{*}(p,s) f(s)d\mu(s) f(p)d\mu(p)d\mu(t)d\mu(z)d\mu(y)\bar{g}(x)d\mu(x))$$

$$= \int\int\int\int\int\int k_{\phi}(x,y)k_{\phi}{}^{*}(y,z)k_{\phi}{}^{n}(z,t)k_{\phi}{}^{n}(t,p)k_{\phi}{}^{*}(p,s) f(s)d\mu(s) f(p)d\mu(p) f(t)d\mu(t) f(z)d\mu(z)$$
$$f(y)d\mu(y) f(x)d\mu(x)$$
...(1)

And $(T_{k\phi}{}^{n}T_{k_{\phi}}{}^{*}T_{k\phi}{}^{*}T_{k\phi}{}^{n}T_{k\phi} f, g) = \int (T_{k\phi}{}^{n}T_{k\phi}{}^{*}T_{k\phi}{}^{*}T_{k\phi}{}^{n}T_{k\phi} f)(x)\bar{g}(x)d\mu(x)$

=

$$= \int\int\int\int\int\int k_{\phi}{}^{n}(x,y)k_{\phi}{}^{*}(y,z)k_{\phi}{}^{*}(z,t)k_{\phi}{}^{n}(t,p)k_{\phi}(p,s) f(s)d\mu(s) f(p)d\mu(p) f(t)d\mu(t) f(z)d\mu(z) f(y)d\mu(y)$$
$$f(x)d\mu(x)$$

…….(2)

It follows from (1) &(2) that $T_{k\phi}$ is quasi n binormal operator.

Conversely suppose that $T_{k\phi}$ is quasi n – binormal take $f = \psi_E$ and $g = \psi_F$ $h = \psi_G$ $I = \psi_J$ we

See that $\int_I\int_E\int_F\int_G k_{\phi}{}^{n}(x,y)k_{\phi}{}^{*}(y,z)k_{\phi}{}^{n}(z,t)k_{\phi}{}^{*}(t,p)k_{\phi}(p,s)d\mu(x)d\mu(y)d\mu(z)d\mu(t)$

$= \int_I\int_E\int_F\int_G k_{\phi}(x,y)k_{\phi}{}^{n}(y,z)k_{\phi}{}^{*}(z,t)k_{\phi}{}^{*}(t,p)k_{\phi}{}^{n}(p,s)$

Hence the required condition holds.

Quasi – n – binormal operators in Fockspace

Let F be a Fock Space and C_ϕ is quasi – n – binormal iff $C_\phi(C_\phi^* C_\phi^n C_\phi^n C_\phi^*) = (C_\phi^* C_\phi^n . C_\phi^n C_\phi^*)C_\phi$

Theorem: 3 Let C_ϕ be a Composition operator on F. Then C_ϕ is quasi - n –binormal if and only if

if and only if $C_\phi(C_\phi^* C_\phi^n C_\phi^n C_\phi^*) = (C_\phi^* C_\phi^n . C_\phi^n C_\phi^*)C_\phi$

Proof:

Consider $C_\phi(C_\phi^* C_\phi^n C_\phi^n C_\phi^*) = C_\phi(M_{K_B} C_{\phi^{(n)} o \tau} . M_{K_B o \phi^{(n)}} C_{\varpi \phi^{(n)}})$

$= M_{K_{Bo\phi}} C_{\phi^n o \varpi \phi} . M_{k_{Bo\phi^{(n)}o\phi}} . C_{\varpi \iota^{(n)} o \phi}$ ....**(1)**

$(C_\phi^* C_\phi^n C_\phi^n C_\phi^*)C_\phi = (M_{K_B} C_\tau C_\phi^n C_\phi^n M_{K_B} C_\tau)C_\phi$

$= (M_{K_B} C_{\phi^{(n)} o \tau} . M_{K_B o \phi^{(n)}} C_{\varpi \phi^{(n)}})C_\phi$

$= M_{K_{Bo\phi}} . C_{\phi^{(n)} o \varpi \phi} . M_{K_B O \phi^{(n)} o \phi} . C_{\varpi \phi^{(n)} o \phi}$ ....**(2)**

Hence the result.

Conclusion

In this paper some basic properties of quasi – n – binormal operators acting on a Hilbert space H are characterized.

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