Design of Robust Controllers for a Continuous Stirred Tank Reactor

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Abstract

During the synthesis of H-infinity and μ controller; closed-loop model of the Continuous Stirred Tank Reactor (CSTR) plant that involves the effect of rate limit due to the actuator dynamics, the uncertainty in the measurements owing to the sensor noises and modeling errors are considered. Performance analysis of the H-infinity and the μ controller revealed that the H-infinity controller was not robust to the model uncertainties. The μ controller was robust when uncertainties occurred and this controller was used to spot the sensitivity to the uncertain element in the temperature and concentration loop.

Keywords: *Rate limit, Actuator dynamics, Model uncertainties, H-infinity and, μ synthesis.*

1. Introduction

CSTR is important in the process equipments; the reactor considered here is exothermic. The notable property of a CSTR is that it has non-linearities such as bifurcations [1], multiple steady states [2], Chaos [3], limit cycle [1] and potential safety issues [4]. CSTR is an extremely non-linear process with multiple inputs and multiple outputs (MIMO) affected by disturbances such as a change in inlet concentration and temperature. The control of the CSTR can be done using the classic PID control where the controller parameters were tuned using the Ziegler Nicholas methods. The position of the Eigen values of the system matrix was used to analyze the stability of the CSTR [5]. A continuous-time adaptive controller was synthesized to maintain the reactor temperature, where the control structure used a linear and a non-linear part. The non-linear part consisted of the simulated steady-state characteristics of the process and the linear part had the solution of the polynomial, whose parameters were selected using pole-placement process [6].

H-infinity control can be constructed using loop shaping methods by dividing the control problem into sensitivity function and complementary function. The synthesized H-infinity controller provided disturbance rejection and robustness against the disturbances [7] [8]. A Takagi-Sugeno (T-S) model of the CSTR was constructed and the H-infinity controller was synthesized to reduce the effects of the disturbances on the inlet concentration and temperature [9]. Model-based control of the pilot plant reactor was built using an artificial intelligence method named Genetic Algorithm (GA). These methods were used to control the reactor temperature covering a broad range with minimal overshoot. The chief disadvantage is the reliance of the control law on the model of the system. For the control to be optimal the model parameters should be known with greater precision [10]. The control law is obtained by solving the differential equations involved in the modeling of the CSTR by using numerical integration techniques namely Euler's method. As the control law is formulated the tuning of the controller parameters is done using the Monte Carlo simulations [11].

The non-linear model of the CSTR is linearized by means of the Feedback linearization so that the linear structured and fixed order controllers can be applied to the model. Two numerical solvers-HIFOO and HINFSTRUCT for structured and fixed-order controller were designed and applied to the CSTR [12][13]. The conventional feedback linearization method involves the linearization of the control point, but the Input/output feedback linearization is considered here. The disadvantage of conventional feedback linearization is when the control point is altered the error will keep on getting altered. The I/O feedback linearization keeps on differentiating the system output until the control input appears in the equation. The simulated CSTR provided a better tracking of the constant signal and the sinusoidal signal [14].

A Robust Static Output Feedback (RSOF) control method was designed for the CSTR with uncertain parameters such as reaction rate constants, reaction enthalpies, heat transfer coefficients. The reactor considered here is a Multi-Input Multi-Output (MIMO); a simple static feedback P controller was designed, here only the measurable states were taken for feedback hence observers were not designed. An optimal controller was constructed for comparison with the designed RSOF in occurrence of parametric uncertainties[15].In some cases, state variables of the reactor are not completely measurable for economic or technical reasons so a T-S based observer was applied to estimate the states and the H-infinity controller was used to ensure robust tracking performance [16],[17].In order to deal with the uncertainties in the reactor the H-infinity control that minimizes the H-infinity norm of the sensitivity function of the nominal system was formulated. The bounds of the uncertainties were fixed within certain range and the uncertain model was built. The H-infinity controller had good tracking performance and disturbance rejection [18].

Parametric uncertainties such as the reaction enthalpies are considered; firstly a robust PI controller was designed for the stabilization of the reactor with uncertain parameters was done. The robust stability region was found using the sixteen plat theorem. The second approach deals with the proposal of H-infinity control technique for decreasing the H-infinity norm by choosing the weights for loop shaping, and the μ synthesis is implemented using numerical method DK iterations [18].

The H-infinity methods can also be used in fault diagnosis; a fault tolerant H-infinity controller robust to actuator faults and external disturbances was developed for Dynamic Positioning Ships based on Sampled-data. The controller was able to track the given signal without steady-state error [19] [20]. The H-infinity control and the backstepping control were applied to the Wheeled Inverted Pendulum (WIP). The dynamic model is separated into two subsystems in order to apply the mentioned controllers. To avoid the instantaneous changes in the states a virtual control is provided [21] [7] [22]. An LMI (Linear Matrix Inequality) which solves the Algebraic Ricatti inequalities rather than solving the Algebraic Ricatti equations was used to design H-infinity problems. Using this method, there builds a connection between controller-free parameters, controller order and closed-loop properties [23].

Organization of the paper is: Section 2 involves the modeling of the CSTR using state-space realization. In Section 3 the closed-loop model of CSTR is handled. Section 4 H-infinity controller design for the CSTR is developed. In Section 5 µ controller synthesis for the CSTR is performed. Lastly, Section 6 presents the conclusions.

2. Modeling of CSTR

Modelling of the CSTR involves using the first principles to obtain the model of a system. Here the species balance and the energy balance of the CSTR is carried out to derive the state-space model and the transfer function model of the system. During the modeling of the CSTR, the following appropriations are made

1. Perfect mixing of the reactants
2. Constant volume of the reactor.

The component mass balance and energy balance principle in the reactor yields the mathematical model equations

The species balance is given as

$$V\frac{dC_a}{dt} = F\left(C_{a,f} - C_a\right) - K_0 \exp\left[\frac{-E}{RT}\right]C_a V \tag{1}$$

The Energy balance is given as

$$V\rho C_p \frac{dT}{dt} = \rho F C_p\left(T_f - T\right) - \Delta H\left[K_0 \exp\left[\frac{-E}{RT}\right]C_a\right]V - UA\left(T_j - T\right) \tag{2}$$

The state-space depiction of the CSTR is given as

$$\begin{bmatrix} \dot{C}_a \\ \dot{T} \end{bmatrix} = \begin{bmatrix} -K_0 \exp\left[\frac{-E}{RT}\right] & 0 \\ -\frac{\Delta H}{\rho C_p} K_0 \exp\left[\frac{-E}{RT}\right] & \frac{UA}{\rho C_p V} \end{bmatrix} \begin{bmatrix} C_a \\ T \end{bmatrix} + \begin{bmatrix} C_{a,f} - C_a & 0 \\ 0 & -\frac{UA}{\rho C_p V} \end{bmatrix} \begin{bmatrix} F/V \\ T_j \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_a \\ T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} C_a \\ T \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} [T_j] \quad (4)$$

Where the reactor concentration (Ca) and temperature (T) are states of the system. Manipulated input is Temperature of the Cooling Jacket (T_j) and dilution rate F/V. Controlled outputs are Concentration of the Reactor (C_a) and Temperature of the Reactor (T). The parameter values of the model are listed.

Table 1. Parameter Values of the Model

Symbol	Parameter	Value
T_j	Temperature of cooling jacket (K)	270
F	Volumetric Flow rate(m^3/sec)	100
V	Volume of CSTR(m^3)	100
P	Density of A-B Mixture(kg/m^3)	1000
C_p	Heat capacity of A-B mixture(J/kg-K)	0.239
ΔH	Heat of reaction for $A \rightarrow B$(J/mol)	5×10^4
E/R	E over R	8750
K_0	Pre-exponential factor(1/sec)	7.2×10^{10}
UA	Overall heat transfer coefficient (U=W/m^2-K)	5×10^4
$C_{a,f}$	Feed concentration (mol/m^3)	1
T	Temperature in CSTR (K)	350
C_a	Concentration in CSTR (mol/m^3)	0.989

3. Building Weighted Closed-Loop Model

3.1 Nominal Plant Model

The CSTR is linearized in the region of the operating points C_a = 0.989 mol/m^3 and T = 296.6 K and the state-space depiction of the nominal plant model is obtained

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

A is $n\times n$ the system matrix, a constant
B is $n\times r$ the input matrix

C is $m \times n$ the output matrix
D is $m \times r$ the direct feed-through
x is $n \times 1$ the state vector
u is $r \times 1$ the input
y is $m \times 1$ the output

$$A = \begin{bmatrix} -0.0285 & -0.0014 \\ -0.0371 & -0.1476 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0850 & 0 \\ 0 & 0.4462 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

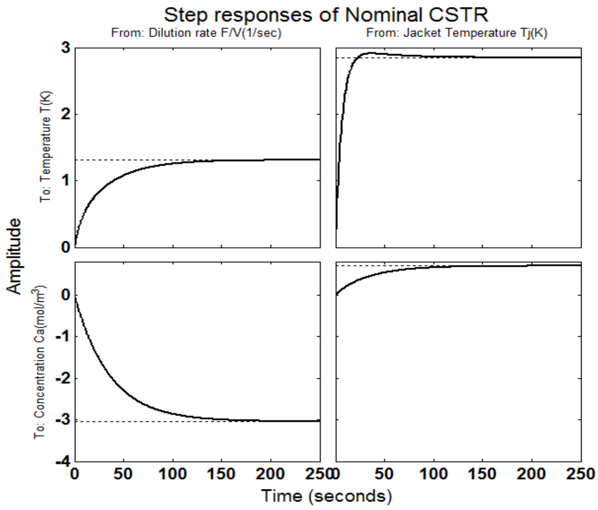

Figure 1. Step Response of the Nominal CSTR Model

3.2 Valve Actuator Dynamics

In general when the closed-loop design is considered the actuator dynamics are not taken into account but this affects the stability of the systems. The actuators are modeled as the first order systems; the rate limit of the actuator affects the signals compared to the pole location hence the actuator dynamics is taken into account. As there are two manipulated inputs there are two actuators and they are modeled as $\frac{20}{s+20}$ and $\frac{20s}{s+20}$. All the measured signals are filtered using a Butterworth filter.

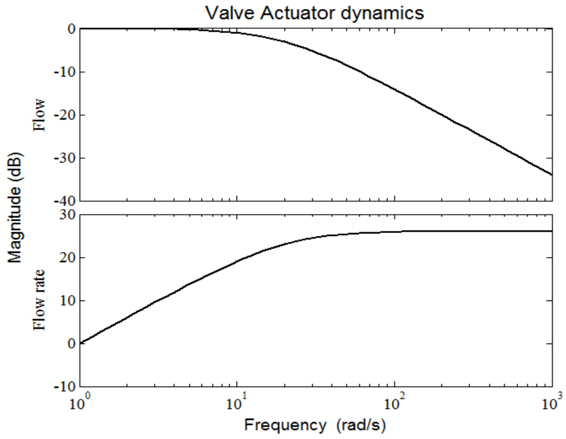

Figure 2. Valve Actuator Dynamics

3.3 Uncertain Model

The mixing and heating effects in the reactor lead to the uncertainty in the response of the measurements of concentration and temperature. Hence the CSTR's uncertain model of has to be constructed. It is not possible to quantify the uncertainty in the model but, it is probable to find the rough boundaries in which the linear model is poor for the particular frequency range.

1. The nominal model of Ca is linear up to 8 Hz and the uncertainties should dominate above 0.09 HZ.
2. The nominal model of T is linear up to 30 Hz and the uncertainties should dominate above 0.005 Hz.

Using this data the rough error bounds on the model is chosen.

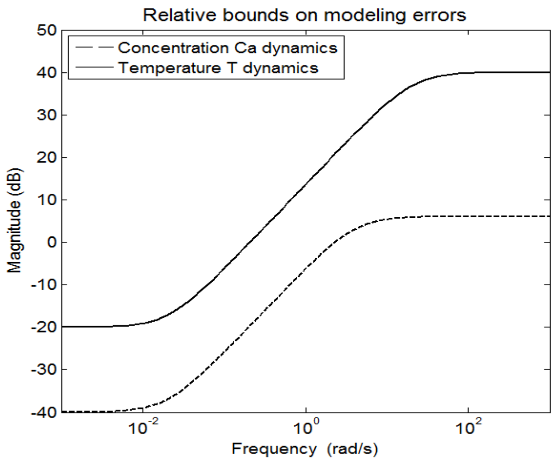

Figure 3. Relative Bounds on Modeling Errors

An uncertain CSTR model was built and the result of the modeling error was simulated for random samples. The model had the effects of the uncertainties when compared to the nominal model.

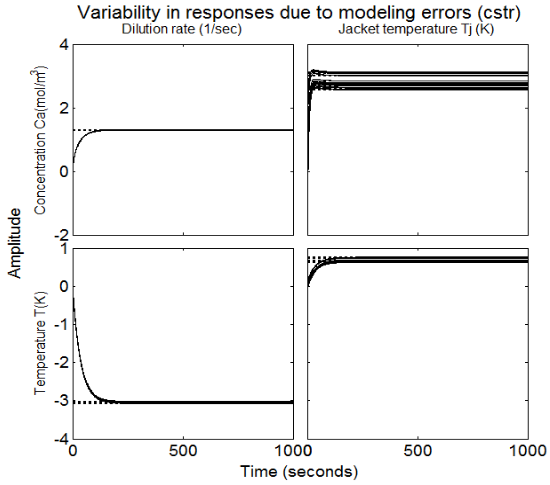

Figure 4. Variability in Response Due To Modeling Errors

3.4 Weighted Closed-Loop Model

The H-infinity algorithm utilizes a closed-loop gain minimization problem that tracks the set-point commands for reactor temperature (T) and reactor concentration (C_a). So suitable weighing functions for sensor noises, set-point commands, tracking errors and actuators have to be chosen. The noise dynamics are selected using FFT analysis. The weights on the sensor dynamics are 0.01 and 0.03. The weight on the error that penalizes the tracking errors on T and Ca and the weights are chosen to be first order low pass filter $\frac{100}{400s+1}$ and $\frac{50}{900s+1}$. The weights on the set-point are chosen to be 0.1 and 0.01 and on the actuator is 0.1 and 50.

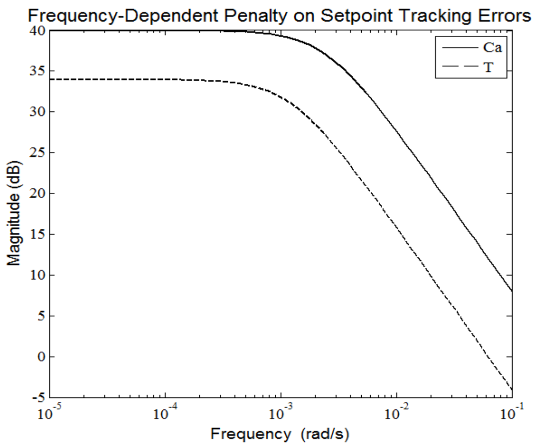

Figure 5. Frequency-Dependent Penalty on Set-Point Tracking Error

Table 2. Weights Being Chosen Using FFT

Weights	Values
WRact(Weight of reactor actuator)	0.1
WJact (Weight of Jacket actuator)	50
WTCmd (Weight of temperature command)	$\frac{100}{400s+1}$
WCaCmd (Weight of concentration command)	$\frac{50}{900s+1}$

WTPerf (Weight of temperature performance)	0.1
WCaPerf (Weight of concentration performance)	0.01
WTNoise (Weight of temperature noise)	0.01
WCaNoise (Weight of concentration noise)	0.03

4. H infinity Controller Synthesis

Once the closed-loop model is built, gain minimization problem can be devised. The gain minimizing control is calculated using the MATLAB function hinfsyn[24]. Minimal value of the closed-loop gain is 0.2807 this shows that the controller satisfies the frequency-domain tracking performance weights. Once the model is simulated in the time-domain the correctness of the performance weights can be checked.

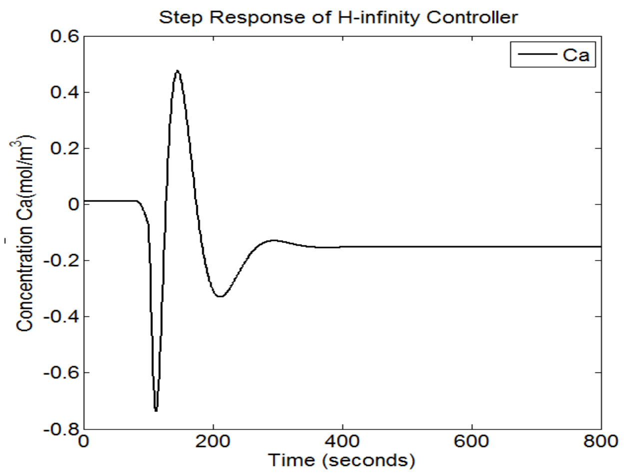

Figure 7. Concentration Response of H-Infinity Controller

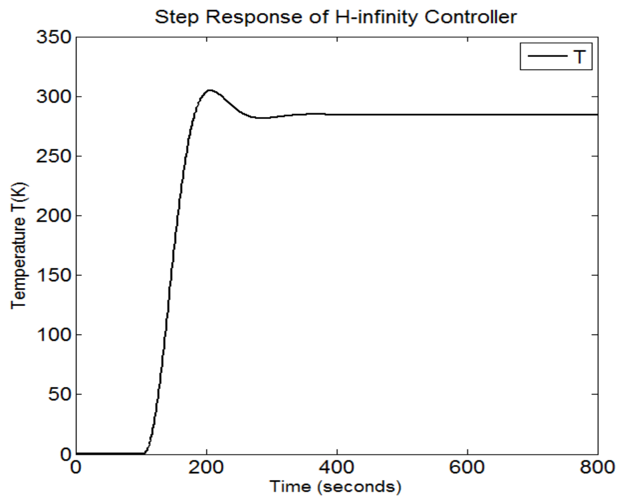

Figure 8. Temperature Response of H-Infinity Controller

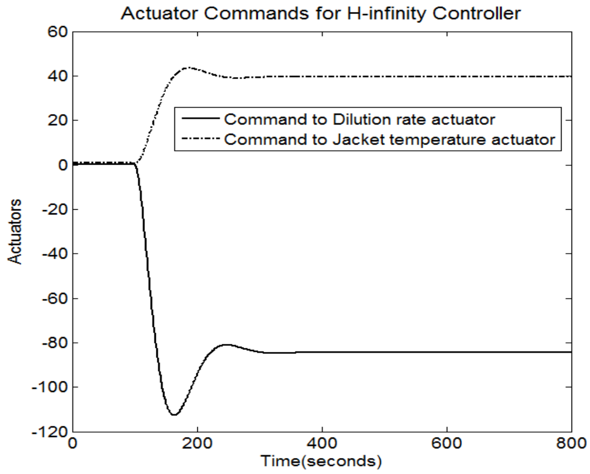

Figure 9. Actuator Commands

H-infinity controller is built for the uncertain model of CSTR. It is likely to compare the closed-loop performance with the worst-case performance considering the model uncertainty set using the MATLAB command *wcgain*.

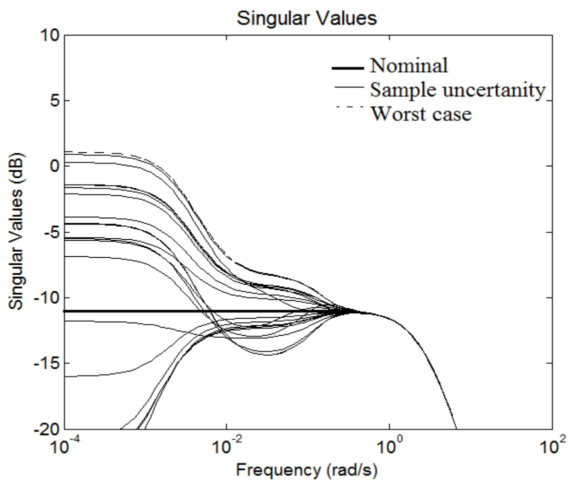

Figure 10. Robustness Analysis Using Singular Values

The worst-case performance of the closed-loop is considerably poorer than the nominal performance which signifies that the H-infinity controller isn't robust to modeling errors.

5. µ Controller Synthesis

The H-infinity controller designed was not robust to model uncertainties hence, the µ controller that uses the D-K iterations were used. The µ controller allows consistent performance for the models with the occurrence of uncertainties and nominal models. The controller synthesis is done using the MATLAB command *dksyn.* The closed-loop model with the controller is simulated in the time-domain as exposed in the figure underneath.

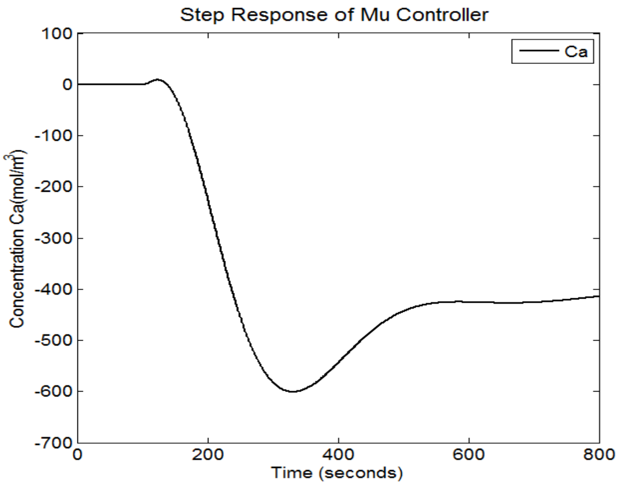

Figure 11. Temperature Response of Mu Controller

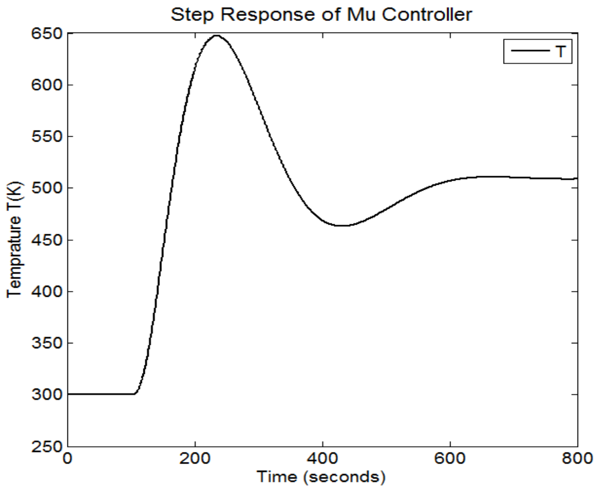

Figure 12. Concentration Response of Mu Controller

Table 3. Comparison of the time-domain performance criteria

Time-domain Specifications	H-infinity		μ controller	
	Ca Profile	T Profile	Ca Profile	T Profile
Rise Time	2.1700	49.7868	0.0112	40.562
Overshoot	175.656	6.9892	1.4010	15.833
Settling Time	236.946	97.1923	318.155	150.96

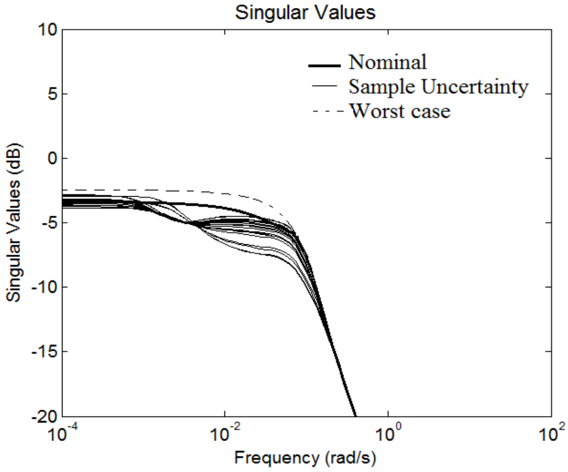

Figure 13. Robustness Analysis Using Singular Values

The worst case gain analysis can be used to calculate the worst-case gain across frequency. Computing the sensitivity to every uncertain element showed that the worst-case peak gain is mainly sensitive to uncertainty in the Temperature loop.

6. Conclusion

The closed-loop model of the Continuous Stirred Tank Reactor (CSTR) plant that includes the valve actuator dynamics, model uncertainties were completed by selecting weights on sensor noises, set-point commands, tracking errors and actuators. The H-infinity controller and the µ controller that were synthesized for the built closed-loop model were compared under the conditions of robustness and concluded that the H-infinity controller wasn't robust to the model uncertainties. The µ controller was robust when uncertainties existed and on using the controller it was found that the sensitivity was high on the uncertainty in temperature loop. The time-domain performance criteria of the H-infinity and the µ controller were obtained and the comparison shows that the µ controller performs poorer compared to the H-infinity controller. But the µ controller performs better compared to the H-infinity controller on condition of robustness to model uncertainties.

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